Chapter 8 HW 2012

Wednesday, October 31, 2012

10:19 PM

112 (a) $_{10}p_{80}^{00} = \frac{l_{90}}{l_{80}} = \frac{1{,}058{,}491}{3{,}914{,}365}$

$= 0.27041$

(b) $_{10}p_{80}^{01} = \frac{l_{80}-l_{90}}{l_{80}} = 1 - 0.27041$

$= .72959$

(c) 0 because a life cannot move from state 1 to state 0.

113 (a)

$$_{10}p_x^{00} = \exp\left(-\int_0^{10}(\mu_x^{01}+\mu_x^{02})\,ds\right)$$

$$= \exp\left(-\int_0^{10} 0.06\,ds\right)$$

$$= e^{-10(.06)} = e^{-0.60}$$

$$= 0.54881$$

(b) $$_{10}p_x^{01} = \int_0^{10} {}_tp_x^{00}\,\mu_{x+t}^{01}\,{}_{10-t}p_{x+t}\,dt$$

$$= \int_0^{10} e^{-0.06t}(0.05)\,e^{-0.02(10-t)}\,dt$$

$$= 0.05\,e^{-0.2}\int_0^{10} e^{-0.06t}\cdot e^{0.02t}\,dt$$

$$= 0.05\,e^{-0.2}\int_0^{10} e^{-0.04t}\,dt$$

$$= 0.05\, e^{-0.2} \int_0^{10} e^{-0.04t}\, dt$$

$$= 0.05\, e^{-0.2} \left. \frac{e^{-0.04t}}{-0.04} \right|_0^{10}$$

$$= \frac{0.05}{0.04} e^{-0.2} \left(1 - e^{-0.4}\right)$$

$$= 0.33740$$

(c) ${}_{10}p_x^{02} = 1 - {}_{10}p_x^{00} + {}_{10}p_x^{01} =$

$1 - 0.54881 - 0.33740 = 0.11379$

(114) (a) ${}_{10}p_{80}^{00} = e^{-\int_0^{10} (\mu_{x+s}^{01} + \mu_{x+s}^{02})\, ds}$

$= e^{-\int_0^{10} (0.05+0.01) ds} = e^{-0.6}$

$= 0.54881$

(b) (i) ${}_0p_x^{00} = 1$

(ii) ${}_0p_x^{10} = 0$

$\Big\}$ since everyone is in state 0 at time zero

(iii) ${}_{1/12}p_x^{00} = {}_0p_x^{00} - {}_0p_x^{00}\left(\frac{1}{12}\right)\left(\mu_x^{01} + \mu_x^{02}\right)$

$+ {}_0p_x^{01}\left(\frac{1}{12}\right)\mu_x^{10}$

$= 1 - (1)\left(\frac{1}{12}\right)(0.06) + (0)\left(\frac{1}{12}\right)(0.03)$

$= 0.995$

(iv) $${}_{1/12}p_x^{01} = {}_0p_x^{01} - {}_0p_x^{01}\left(\tfrac{1}{12}\right)\left(\mu_x^{10}+\mu_x^{12}\right) + {}_0p_x^{00}\left(\tfrac{1}{12}\right)\left(\mu_x^{01}\right)$$

$$= 0 - 0 + (1)\left(\tfrac{1}{12}\right)(.05) =$$

$$= 0.004167$$

(v) $${}_{2/12}p_x^{00} = {}_0p_x^{00} - {}_0p_x^{00}\left(\tfrac{1}{12}\right)\left(\mu_x^{01}+\mu_x^{02}\right) + {}_0p_x^{01}\left(\tfrac{1}{12}\right)\left(\mu_x^{10}\right)$$

$$= 0.995 - .995\left(\tfrac{1}{12}\right)(.06) + 0.004167\left(\tfrac{1}{12}\right)(.03)$$

$$= 0.990035$$

(vi) $${}_{2/12}p_x^{01} = {}_{1/12}p_x^{01} - {}_{1/12}p_x^{01}\left(\mu_x^{10}+\mu_x^{12}\right) + {}_{1/12}p_x^{00}\left(\tfrac{1}{12}\right)\left(\mu_x^{01}\right)$$

$$= .00416\bar{6}\left(1-\left(\tfrac{1}{12}\right)(0.05)\right) + .995\left(\tfrac{1}{12}\right)(0.05)$$

$$= 0.008295$$

(115) $${}_{\frac{121}{12}}p_x^{00} = {}_{10}p_x^{00} - {}_{10}p_x^{00}\left(\tfrac{1}{12}\right)\left(\mu_x^{01}+\mu_x^{02}\right) + {}_{10}p_x^{01}\left(\tfrac{1}{12}\right)\left(\mu_x^{10}\right)$$

$$= .9 - .9\left(\tfrac{1}{12}\right)(.06 + 0.01) + 0.07\left(\tfrac{1}{12}\right)(.025)$$

$$= 0.895041667$$

$${}_{\frac{121}{12}}p_x^{01} = {}_{10}p_x^{01}\left(1-\left(\tfrac{1}{12}\right)\left(\mu_x^{10}+\mu_x^{12}\right)\right)$$

$$+ \; {}_{10}p_x^{00}\left(\tfrac{1}{12}\right)\left(\mu_x^{01}\right)$$

$$= .07\left(1-\left(\tfrac{1}{12}\right)(0.025+0.02)\right)$$

$$+ (.9)\left(\tfrac{1}{12}\right)(0.06)$$

$$= 0.074238$$

(116) Pr (Transition from 1 to 0 at time t but then not remaining continuously in 0 until 5)

$$= \int_0^5 {}_tp_x^{\overline{11}} \; \mu_{x+t}^{10} \left(1 - {}_{5-t}p_{x+t}^{\overline{00}}\right) dt$$

$$= \int_0^5 \exp\left[-\int_0^t (\mu_x^{10} + \mu_x^{12})\, ds\right] \cdot \mu^{10} \cdot \left[1 - \exp\left[-\int_t^5 (\mu_x^{01} + \mu_x^{02}) ds\right]\right] dt$$

$$= 0.03 \int_0^5 \exp\left[-\int_0^t 0.07\right]\left[1-\exp\left[-\int_t^5 0.07\right]\right] dt$$

$$= 0.03 \int_0^5 e^{-0.07t}\left[1 - e^{-0.07(5-t)}\right] dt$$

$$= 0.03 \int_0^5 \left(e^{-0.07t} - e^{-0.35}\right) dt$$

$$= 0.03\left[\frac{1-e^{-0.35}}{0.07} - 5e^{-0.35}\right]$$

$$= 0.03(4.2187 - 3.5234) = \boxed{0.0209}$$

(117) The probability that Emily

will be an executive employee at age 55 is

$${}_{30}p_{25}^{01} \cdot Pr(\text{surviving } 30 \text{ years})$$

$$Pr(\text{surviving } 30 \text{ years}) = 0.92$$

$${}_{30}p_{25}^{01} = \int_0^{30} {}_tp_{25}^{00} \, \mu_{25+t}^{01} \, {}_{30-t}p_{25+t} \, dt$$

$$= \int_0^{30} e^{-\int_0^t (\mu^{01}+\mu^{02})ds} \cdot \mu^{01} \cdot e^{-\int_t^{30} \mu^{12} ds} \, dt$$

$$= \int_0^{30} e^{-\int_0^t 0.028\,ds} (0.008)\, e^{-\int_t^{30} 0.01\,ds} \, dt$$

$$= \int_0^{30} e^{-0.028t} (0.008)\, e^{-0.01(30-t)} \, dt$$

$$= (0.008)\, e^{-0.30} \int_0^{30} e^{-0.018t} \, dt$$

$$= \frac{0.008}{0.018} e^{-0.30} \left(1 - e^{-0.54}\right)$$

$$= 0.13738$$

$$ANS = (0.13738)(.92) = \boxed{0.12639}$$

118

STATE 0 → IN GOOD STANDING

1 → OUT OF FAVOR

2 → DEAD

Using a tree approach

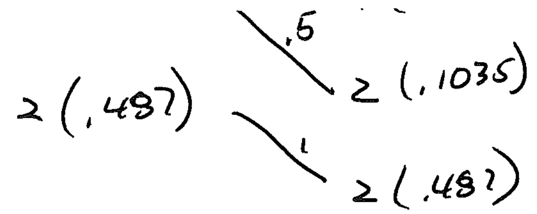
TIME 0	TIME 1	TIME 2	REGROUP
0	$\xrightarrow{.6}$ 0 (.6)	$\xrightarrow{.6}$ 0 (.36)	0 (.42)
		$\xrightarrow{.3}$ 1 (.18)	1 (.27)
		$\xrightarrow{.1}$ 2 (.06)	2 (.31)
	$\xrightarrow{.3}$ 1 (.3)	$\xrightarrow{.2}$ 0 (.06)	
		$\xrightarrow{.3}$ 1 (.09)	
		$\xrightarrow{.5}$ 2 (.15)	
	$\xrightarrow{.1}$ 2 (.1)	$\xrightarrow{1}$ 2 (.1)	

TIME 2

		REGROUP TIME 3
0 (.42)	$\xrightarrow{.6}$ 0 (0.252)	0 (0.306)
	$\xrightarrow{.3}$ 1 (0.126)	
	$\xrightarrow{.1}$ 2 (0.042)	1 (0.207)
1 (.27)	$\xrightarrow{.2}$ 0 (.054)	
	$\xrightarrow{.3}$ 1 (0.081)	
	$\xrightarrow{.5}$ 2 (.135)	
2 (.31)	$\xrightarrow{1}$ 2 (.31)	2 (0.487)

TIME 3

		TIME 4
0 (.306)	$\xrightarrow{.6}$ 0 (0.1836)	0 (0.225)
	$\xrightarrow{.3}$ 1 (0.0918)	1 (0.1539)
	$\xrightarrow{.1}$ 2 (0.0306)	2 (.6211)
1 (.207)	$\xrightarrow{.2}$ 0 (0.0414)	
	$\xrightarrow{.3}$ 1 (0.0621)	
	$\xrightarrow{.5}$ 2 (.1035)	
2 (.487)		



ANSWER = 15.39%

(119) (i) see above

$1000(0.6211) = 621.1$

(ii) BINOMIAL SO

$N(p)(q) = 1000(.6211)(1-0.6211)$

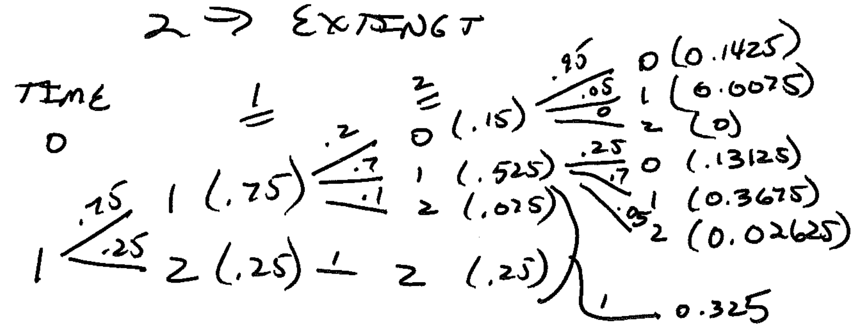
$= 235.33479$

(120) STATES

0 ⇒ HEALTHY

1 ⇒ ENDANGERED

2 ⇒ EXTINCT



REGROUP AT END OF 3 YEARS

0 0.27375

1 0.375

2 0.35125

Note under Q i nobody becomes extinct so answer is <u>0.35125</u>

121

$$A_x^{01} = \int_0^\infty v^t \, {}_tp_x^{00} \, \mu_{x+t}^{01} \, dt$$

$$= \int_0^\infty e^{-0.045t} e^{-0.115t} (.015) \, dt$$

$$= \frac{.015}{.16} = \frac{15}{160} = \frac{3}{32}$$

$$\ddot{a}_x^{00} = \int_0^\infty v^t \, {}_tp^{00} \, dt =$$

$$\int_0^\infty e^{-0.04t} e^{-0.115t} \, dt$$

$$= \frac{1}{0.16}$$

$$P = \frac{1000\left(\frac{0.015}{.16}\right)}{\frac{1}{.16}} = 1000(0.015) = \boxed{15}$$

122

$$A_x^{01} = \int_0^\infty v^t \, {}_tp_x^{00} \, \mu_{x+t}^{01} \, dt$$

$$= \int_0^\infty e^{-0.06t} e^{-0.04t} (.015) \, dt$$

$$= \frac{.015}{.10}$$

$$A_x^{02} = \int_0^\infty v^t \, {}_tp_x^{00} \, \mu_{x+t}^{02} \, dt$$

$$= \int_0^\infty e^{-\delta t} e^{-0.04t} (.025)\, dt$$

$$= \frac{.025}{.10}$$

$$\bar{a}_x^{00\,00} = \int_0^\infty v^t \, {}_t p_x^{00} \, dt =$$

$$\int_0^\infty e^{-0.06t} e^{-0.04t} \, dt$$

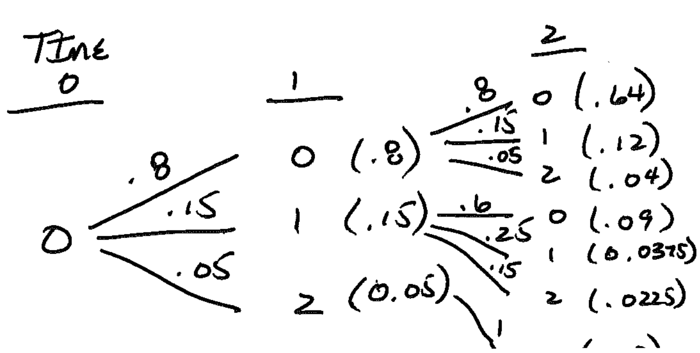
$$= \frac{1}{.10}$$

$$P = \frac{2000\left(\frac{0.015}{.10}\right) + 1000\left(\frac{0.025}{.10}\right)}{\frac{1}{.10}}$$

$$= 2000(0.015) + 1000(0.025)$$

$$= 30 + 25 = \boxed{55}$$

(123) First, note that everyone is healthly at time zero.



$\searrow$ 2 (.05)

REGROUP

0 (0.73) $\xrightarrow{.8}$ 0 (.584)

$\xrightarrow{.15}$ 1 (.1095

$\xrightarrow{.05}$ 2 (.0365)

1 (.1575) $\xrightarrow{.6}$ 0 (0.0945)

$\xrightarrow{.25}$ 1 (0.039375)

$\xrightarrow{.15}$ 2 (0.023625

2 (.1125) $\longrightarrow$ 2 (.1125)

REGROUP

0 (.6785)

1 (.148875)

2 (.172625)

(i) PV of DB =

$$500{,}000\left(.05v + (.1125-0.05)v^2 + (.172625-.1125)v^3\right)$$

$$= 71{,}140.12$$

(ii) PV of DIS

$$= 100{,}000\left(.15v + (.1575)v^2 + (.148875)v^3\right)$$

$$= 37838.09$$

(iii) PVP = PVB

$$PVP = P\left(1 + (.8)v + (.73)v^2 = 2.33058\right)$$

$$PVB = 71{,}140.12 + 37838.09$$

$$P = \frac{71{,}140.12 + 37838.09}{2.33058}$$

$$= 46{,}760.15$$

(iv) $\quad {}_1V = \underline{({}_0V + P_0)(1+i) - \text{Benefits Pd}}$

$$\overline{\quad Pr(Alive) \quad}$$

$$= \frac{(0 + 46{,}760.15)(1.1) - (100{,}000)(.15) - 500000(.05)}{1 - 0.05}$$

$$= 12{,}038.07$$

(V) $_1V = PVFB - PVFP$

STATE	1	2	3
0	1.00	0.8	0.73
1		0.15	0.1575
2		0.05	0.1125

Note: Since these are Homogeneous Markov chains, we can use values from part I.

$$500{,}000\left[0.05v + (.1125 - 0.05)v^2\right]$$

$$+ 100{,}000\left[0.15v + 0.1575v^2\right]$$

$$- 46{,}760.15\left[1 + .8v\right]$$

$$= -5560.92$$

(VI)

STATE	1	2	3
0		0.60	0.63
1	1.00	0.25	0.1525
2		0.15	0.2175

These can be developed from tree

$PVFB - PVFP =$

$500{,}000\left(.15v + (.2175-.15)v^2\right) +$

$100{,}000\left(0.25v + (0.1575)v^2\right) -$

$46760.15\left[0 + .6v\right] =$

$105{,}899.42$

Note Relationship between iv, v, and vi

$$\frac{(.8)(-5560.92)+(.15)(105{,}899.42)}{.95}$$

$= 12038.08$

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STATE	TIME 0	1	2	3	4
0	1	.6	.42	.306	.225
1		.3	.27	.207	.1539
2		.1	.31	.487	.6211

From PROBLEM 118

(i) $APV = 10000\left[(.1)(v)+(.21)v^2+(.177)(v^3)+(.1341)v^4\right]$

$= 3599.5136$

(ii) PVP = PVB

$P\left[1+0.9v+0.69v^2+0.513v^3\right] = 3599.5136$

$P = 1484.79$

(iii) Use recursive formula

(iii) Use recursive formula

$$ {}_1V = \frac{({}_0V + P)(1+i) - \text{Benefits Pd}}{1 - \text{pr(dead)}} $$

$$ = \frac{(0 + 1484.79)(1.25) - (.1)(10000)}{.9} = $$

$$ = 951.10 $$

$$ {}_2V = \frac{(951.10 + 1484.79)(1.25) - 10{,}000(.21/.9)}{(.69)/.9} $$

$$ = 928.08 $$

(iv)

STATUS	0	1	2	3	4
0		.2	0.18	0.138	0.1026
1	1.00	.3	0.15	0.099	0.0711
2		.5	0.67	0.763	0.8263

TIME

$$ PVB = 10{,}000 \left[(.5)v + .17v^2 + .093v^3 + 0.0633v^4 \right] $$

$$ = 5823.44 $$

(v) $PVP - PVB =$

$$ 3500 \left[1 + .5v + .33v^2 + .237v^3 \right] - 5823.44 $$

$$ = 240.46 $$

(125)

STATE	TIME 0	1	2	3	4
0	1	.6	.42	.306	.225
1		.3	.27	.207	.1539
2		.1	.31	.487	.6211

FROM PROBLEM (118)

(i) $100{,}000\left[.6v + .42v^2 + .306v^3 + .225v^4\right] +$

$50000\left[.3v + .27v^2 + .207v^3 + .1539v^4\right]$

$= 128{,}854.27$

(ii) PVP = PVB

$P\left[1 + .9v + .69v^2 + .513v^3\right] =$

$128{,}854.27$

$$P = \frac{128{,}854.27}{2.424286} = 53{,}152.09$$

(iii) Using Recursion

$$_1V = \frac{(0 + 53{,}152.09)(1.25) - 100{,}000(.6) + 50000(.3)}{.9} = -9510.99$$

$$_2V = \frac{(-9510.99 + 53{,}152.09)(1.25) - (100{,}000)(.42/.9) - 50000(.27/.9)}{.69/.90} = -9280.82$$

(126) (a) See table in answers

(b) (i) $_{3}p_{55}^{(\tau)} = \frac{l_{58}^{(\tau)}}{l_{55}^{(\tau)}} = \frac{6605.1}{}$

(b) (i) $_3p_{55}^{(\tau)} = \dfrac{l_{58}}{l_{55}^{(\tau)}} = \dfrac{6605.1}{10{,}000}$

$= \boxed{.66051}$

(ii) $_2q_{56}^{(2)} = \dfrac{d_{56}^{(2)} + d_{57}^{(2)}}{l_{56}^{(\tau)}}$

$= \dfrac{492 + 293.56}{8200}$

$= \boxed{0.0958}$

(iii) $_{1|2}q_{55}^{(3)} = \dfrac{d_{56}^{(3)} + d_{57}^{(3)}}{l_{55}}$

$= \dfrac{123 + 146.78}{10{,}000} = \boxed{0.026978}$

(iv) $\dfrac{d_{55}^{(1)} + d_{56}^{(1)} + \cdots + d_{59}^{(1)} + d_{55}^{(3)} + d_{56}^{(3)} + \cdots + d_{59}^{(3)}}{l_{55}^{(\tau)}}$

$= \dfrac{2136.763305}{10000}$

$\boxed{= 0.21368}$

(c) (i) $_{0.25}q_{55}^{(2)} = (0.25)(q_{55}^{(2)})$

$= (0.25)\left(\frac{1500}{10{,}000}\right) = 0.0375$

(ii) $_{0.5}p_{56}^{(T)} = 1 - {}_{0.5}q_{56}^{(T)}$

$= 1 - (0.5)\,q_{56}^{(T)} = 1 - (0.5)(0.105)$

$= 0.9475$

(iii) $_{0.5}p_{56.8}^{(T)} = \dfrac{l_{57.3}^{(T)}}{l_{56.8}^{(T)}}$

$= \dfrac{(.7)(7339) + (.3)(6605.1)}{(.2)(8200) + (.8)(7339)}$

$= \dfrac{7118.83}{7511.20} = \boxed{0.94776}$

(iv) $_{0.5}q_{55.6}^{(1)} = \dfrac{0.4\,d_{55.6}^{(1)} + 0.1\,d_{56}^{(1)}}{l_{55.6}^{(T)}}$

$= \dfrac{(0.4)(200) + (0.1)(246)}{(.4)(10000) + (.6)(8200)}$

$= \dfrac{104.6}{8920} = \boxed{0.01173}$

(d) (i) $_{0.25}q_{55}^{(2)} = \dfrac{q_{55}^{(2)}}{q_{55}^{(T)}}\left(1 - \left(p_{55}^{(T)}\right)^{0.25}\right)$

$$= \frac{0.15}{0.18}\left(1-(.82)^{1/4}\right) = \boxed{0.04034}$$

(ii) $${}_{0.5}p_{56}^{(T)} = \left(p_{56}^{(T)}\right)^{1/2} =$$

$$(0.895)^{1/2} = \boxed{0.94604}$$

(iii) $${}_{.5}p_{56.8}^{(T)} = \frac{l_{57.3}^{(T)}}{l_{56.8}^{(T)}}$$

$$= \frac{l_{57} \cdot {}_{.3}p_{57}^{(T)}}{l_{56} \cdot {}_{.8}p_{56}^{(T)}} = \frac{7339(0.9)^{0.3}}{8200(.895)^{.8}}$$

$$= \boxed{0.94763}$$

(iv) $${}_{0.5}q_{55.6}^{(1)} = \frac{{}_{0.4}d_{55.6}^{(1)} + {}_{0.1}d_{56}^{(1)}}{l_{55.6}^{(T)}}$$

$$= \frac{d_{55}^{(1)} - {}_{0.6}d_{55}^{(1)} + {}_{.1}d_{56}^{(1)}}{l_{55}\left(p_{55}^{(T)}\right)^{.6}}$$

$$= \frac{200 - 10000\left(\frac{.02}{.18}\right)\left(1-(.82)^{.6}\right) + 8200\left(\frac{.03}{.105}\right)\left(1-(.895)^{.1}\right)}{10{,}000\,(.82)^{.6}}$$

$$= \frac{101.1184537}{} = \boxed{0.011390}$$

$\overline{8877.45156}$

(127)

x	$q_x^{(1)}$	$q_x^{(2)}$	$p_x^{(T)}$	$l_x^{(T)}$	$d_x^{(1)}$	$d_x^{(2)}$
20	0.03	0.01	.96	1000	30	10
21	0.025	0.02	.955	960	24	19.2
22	0.020	0.03	.95	916.8	18.336	27.504

PVP = PVB

$$P(1000 + 960v + 916.8v^2)$$

$$= 2000(30v + 24v^2 + 18.336v^3)$$

$$+ 1000(10v + 19.2v^2 + 27.504v^3)$$

$P = 63.64$

${}_0V = 0$ by definition

${}_3V = 0$ by defintion

$${}_1V = \frac{({}_0V + P)(1+i) - b_1^{(1)} q_x^{(1)} - b_1^{(2)} q_x^{(2)}}{p_x^{(T)}}$$

$$= \frac{(0 + 63.64)(1.1) - (2000)(0.03) - 1000(0.01)}{.96}$$

$$= \boxed{0.00}$$

$$2V = \frac{(0.00 + 63.64)(1.1) - 2000(0.025) - 1000(.02)}{.955}$$

$$= 0.00$$

(128) $q_x^{(1)} = q_x^{\prime(1)}\left\{1 - \tfrac{1}{2}\left[q_x^{\prime(2)} + q_x^{\prime(3)}\right] + \tfrac{1}{3} q_x^{\prime(2)} q_x^{\prime(3)}\right\}$

$$= (.2)\left\{1 - \tfrac{1}{2}(0.08 + 0.125) + \tfrac{1}{3}(0.08)(0.125)\right\}$$

$$= \boxed{0.180167}$$

(129) $p_x^{(\tau)} = p_x^{\prime(1)} \cdot p_x^{\prime(2)} \cdot p_x^{\prime(3)}$

$$= (1 - q_x^{\prime(1)})(1 - q_x^{\prime(2)})(1 - q_x^{\prime(3)})$$

$$= (0.8)(0.92)(0.875)$$

$$= 0.644$$

$${}_1p_x^{\prime(1)} = \left(p_x^{(\tau)}\right)^{\frac{q_x^{(1)}}{q_x^{(\tau)}}}$$

$$0.8 = (0.644)^{\frac{q_x^{(1)}}{0.356}} \Rightarrow q_x^{(1)} = \frac{\ln(0.8)}{\ln(0.644)}(0.356)$$

$$= \boxed{0.180520}$$

(130) $q_x^{\prime(1)} = 0.200$

$$q_x'^{(2)} = 0.080$$

$$p_x^{(T)} = p_x'^{(1)} \cdot p_x'^{(2)} = (1-q_x'^{(1)})(1-q_x'^{(2)})$$

$$= (.8)(.92) = .736$$

$$q_x'^{(1)} = 1 - \left(p_x^{(T)}\right)^{\frac{q_x^{(1)}}{q_x^{(T)}}}$$

$$.8 = 1 - (.736)^{\frac{q_x^{(1)}}{.264}}$$

$$\therefore q_x^{(1)} = 0.192186173$$

$$_{0.4}q_{x+0.4} = \frac{_{0.4}d^{(1)}_{x+0.4}}{l^{(T)}_{x+0.4}}$$

$$= \frac{(0.4)(0.192186173)}{1-0.4q_x^{(T)}}$$

$$= \frac{(0.4)(0.192186173)}{1-0.4(1-0.736)} = \boxed{0.085951}$$

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$$q_x^{(2)} = \frac{d_x^{(2)}}{l_x^{(T)}}$$

Let $l_x^{(T)} = 1000$

$$d_x^{(1)} = l_x^{(T)} \cdot q_x'^{(1)} = 1000(.2) = 200$$

Since this is uniformly distributed this means that $(0.6)(200)$ is the number that occur before time 0.6. Therefore 120 lives leave from cause 1 in the first .6 of a from cause 1 in the first .6 of a year leaving us with 880 at time 0.6

$$d_x^{(2)} = 880\left(q'^{(2)}\right) = 880(.08) = 70.4$$

so $q_x^{(2)} = \dfrac{70.4}{1000} = \boxed{0.0704}$

132 $p_{40}^{(T)} = 1 - q_{40}^{(1)} - q_{40}^{(2)} = p_{40}'^{(1)} \cdot p_{40}'^{(2)}$

$= 1 - 0.24 - 0.1 = 0.66$

$= (1-0.25)(1-y)$

$\therefore\ y = 1 - \dfrac{0.66}{0.75} = 0.12$

$\therefore\ 2y = 0.24$

$l_{42}^{(T)} = l_{40}^{(T)} \cdot p_{40}^{(T)} \cdot p_{41}^{(T)}$

$= l_{40}^{(T)} \cdot p_{40}'^{(1)} \cdot p_{40}'^{(2)} \cdot p_{41}'^{(1)} \cdot p_{41}'^{(2)}$

$= 2000(1-0.25)(1-0.12)(1-0.2)(1-0.24)$

$= \boxed{802.56}$

133

Let (M) ⇒ Mechanical Failure

Let (O) ⇒ Other Reasons

Build a table Start with 1000 phones

YR. OF SERVICE	$d_t^{(m)}$	$d_t^{(o)}$	Phones that Survive
1	200 ①	300 ②	500 ③

2	50 ⑥	200 ④	250 ⑤
3	100 ⑨	100 ⑦	50 ⑧

① $1000(q_1^{(m)}) = 1000(.2) = 200$

② $1000(q_1^{(o)}) = 1000(.3) = 300$

③ $1000 - 200 - 300 = 500$

④ $500(q_2^{(o)}) = 500(.4) = 200$

⑤ From ⑥ $200 = (.8)(⑤)$

$⑤ = 250$

⑥ $500 - 200 - 250 = 50$

⑦ From ⓐ $⑦ = (.40)(250) = 100$

$⑧ = 250(.2) = 50$

⑨ $250 - 100 - 50 = 100$

$$_3q_x^{(m)} = \frac{d_1^{(m)} + d_2^{(m)} + d_3^{(m)}}{\ell_0^{(\tau)}} =$$

$$\frac{200 + 50 + 100}{1000} = 0.35$$

134 - 141 see next page

(142)

ⓐ $_tp_x = e^{-\int_0^t \mu_{x+s}\, ds}$

134

Derive independent $p'^{(d)}_{60} = p^{00\left(p^{0d}_{60}/p^{0*}_{60}\right)}_{60}$

$$\text{where } p^{0*}_{60} = p^{0d}_{60} + p^{0w}_{60} = 1 - \frac{l^{(\tau)}_{61}}{l^{(\tau)}_{60}}$$

$$= \left(l^{(\tau)}_{61} / l^{(\tau)}_{60}\right)^{(2{,}580/950{,}000)/\left(1-\ell^{(\tau)}_{61}/\ell^{(\tau)}_{60}\right)}$$
$$= 0.897556^{(0.002716/0.102444)} = 0.997139$$

So $p^{0w}_{60} = \log p'^{(w)}_{60} / \log p^{00}_{60} \times \left(1 - p^{00}_{60}\right)$
$= \log 0.95 / \log \left[(0.997139)(0.95)\right] \times (1 - (0.997139)(0.95)) = 0.049929$

So $d^{(w)}_{60} = 950{,}000 \times 0.049929 = 47{,}433$

(135)

(a) $${}_{10}q_{50:60} = 1 - {}_{10}p_{50:60} = 1 - {}_{10}p_{50} \cdot {}_{10}p_{60}$$

$$= 1 - \frac{l_{60}}{l_{50}} \cdot \frac{l_{70}}{l_{60}} = 1 - \frac{l_{70}}{l_{50}}$$

$$= 1 - \frac{6{,}616{,}155}{8{,}950{,}901} = 0.26084$$

(b) $${}_{10|}q_{50:60} = {}_{10}p_{50:60}\,(1 - p_{60:70})$$

$$= (1 - 0.26084)\left(1 - \frac{l_{61}}{l_{60}} \cdot \frac{l_{71}}{l_{70}}\right)$$

$$= (1 - 0.26084)\left(1 - \frac{8{,}075{,}403}{8{,}188{,}074} \cdot \frac{6{,}396{,}609}{6{,}616{,}155}\right)$$

$$= 0.03436$$

(c) $A_{60:60} = .47975$

(D) $$P_{60:60} = \frac{A_{60:60}}{\ddot{a}_{60:60}} = \frac{.47975}{9.1911} = 0.05220$$

(E) $$\bar{A}_{60:60} = \frac{i}{\delta} A_{60:60} = (1.02971)(.47975) = 0.49400$$

(f) $$P(\bar{A}_{60:60}) = \frac{\bar{A}_{60:60}}{\bar{a}_{60:60}} = \frac{\frac{i}{\delta} A_{60:60}}{\frac{1 - \frac{i}{\delta} A_{60:60}}{\delta}} = \frac{(1.02971)(.47975)}{8.68381}$$

$$= 0.056887$$

$$\times 1000$$

$$= \underline{\underline{56.89}}$$

135

(g) $${}_{10}p_{\overline{50:60}} = {}_{10}p_{50} + {}_{10}p_{60} - {}_{10}p_{50:60}$$

$$= \frac{l_{60}}{l_{50}} + \frac{l_{70}}{l_{60}} - \frac{l_{60}}{l_{50}}\frac{l_{70}}{l_{60}}$$

$$= \frac{8,188,074}{8,950,901} + \frac{6,616,155}{8,188,074} - \frac{6,616,155}{8,950,901}$$

$$= .98364$$

(h) $${}_{10}p_{\overline{50:60}} - {}_{11}p_{\overline{50:60}}$$

$$= .98364 - ({}_{11}p_{50} + {}_{11}p_{60} - {}_{11}p_{50}\,{}_{11}p_{60})$$

$$= .98364 - \left[\frac{8,075,403}{8,950,901} + \frac{6,396,609}{8,188,074} - \frac{8,075,403}{8,950,901}\frac{6,396,609}{8,188,074}\right]$$

$$= .98364 - .97860 = 0.00504$$

(i) $${}_{10}p_{\overline{50:60}} - {}_{10}p_{50:60} = 0.98364 - [1 - .26084] = 0.24448$$

(j) $$A_{\overline{60:70}} = A_{60} + A_{70} - A_{60:70}$$

$$= .36913 + .51495 - .57228$$

$$= 0.31180$$

(k) $$P_{\overline{60:70}} = \frac{A_{\overline{60:70}}\,(d)}{1 - A_{\overline{60:70}}} = \frac{0.31180\left(\frac{.06}{1.06}\right)}{1 - 0.31180} = 0.02565$$

$\times 1000$

$\underline{\underline{25.65}}$

(l) $$\ddot{a}_{60} + \ddot{a}_{60} - \ddot{a}_{60:60} = (2)(11.1454) - 9.1911$$

$$= 13.0997$$

(135) (m) $\ddot{a}_{\overline{60:70}} = \ddot{a}_{60} + \ddot{a}_{70} - \ddot{a}_{60:70}$

$= 11.1454 + 8.5693 - 7.5563$

$= 12.1584$

or $\dfrac{1 - A_{\overline{60:70}}}{d} = \dfrac{1 - .31180}{\frac{.06}{1.06}} = 12.1582$

(n) $\ddot{a}_{70|60} = \ddot{a}_{60} - \ddot{a}_{60:70}$

$= 11.1454 - 7.5563 = 3.5891$

(136)

$a\ \ddot{a}_{50} + b\ \ddot{a}_{60} + c\ \ddot{a}_{50:60}$

$a = 2/3 \qquad b = 2/3 \qquad c = 1 - a - b = -1/3$

$2/3(13.2668) + 2/3(11.1454) - 1/3(10.1944)$

$= 12.8767$

(137)

$a\ \ddot{a}_{50} + b\ \ddot{a}_{60} + c\ \ddot{a}_{50:60}$

$a = 2/3 \qquad b = 1/2 \qquad c = 1 - 2/3 - 1/2 = -\frac{1}{6}$

$2/3\ (13.2668) + 1/2(11.1454) - 1/6\ (10.1944)$

$= 12.7182$

138

$$l_{90:91} = (1000)(900) = 900{,}000$$

$$l_{91:92} = (900)(720) = 648{,}000$$

$$l_{92:93} = (720)(432) = 311{,}040$$

$$l_{93:94} = (432)(216) = 93{,}312$$

$$l_{94:95} = (216)(0) = 0$$

(a)

$$A_{91:92} = \frac{(648{,}000 - 311{,}040)\left(\frac{1}{1.04}\right) + (311{,}040 - 93{,}312)\left(\frac{1}{1.04}\right)^2 + (93312)\left(\frac{1}{1.04}\right)^3}{648{,}000}$$

$$= \underline{\underline{0.93867}}$$

(b)

$$A^{\;\;1}_{\overline{90:91}:\overline{3|}} = \frac{(900{,}000 - 648{,}000)\left(\frac{1}{1.04}\right) + (648{,}000 - 311040)\left(\frac{1}{1.04}\right)^2 + (311{,}040 - 93{,}312)\left(\frac{1}{1.04}\right)^3}{900{,}000}$$

$$= 0.83045$$

(c)

$$\ddot{a}_{92:93} = \frac{311{,}040 + 93{,}312\left(\frac{1}{1.04}\right)}{311{,}040} = \underline{\underline{1.28846}}$$

(139) $_6p_{40:40} - {}_{12}p_{40:40} = {}_6p_{40}\,{}_6p_{40} - {}_{12}p_{40}\,{}_{12}p_{40}$

$= (.98)^2 - (.945)^2 = 0.067375$

(140)

$PV = v^5\left({}_4p_{\overline{70:80}} - {}_5p_{\overline{70:80}}\right)(10{,}000)$

$= v^5\left[1 - {}_4q_{\overline{70:80}} - \left(1 - {}_5q_{\overline{70:80}}\right)\right](10{,}000)$

$= v^5\left[{}_5q_{\overline{70:80}} - {}_4q_{\overline{70:80}}\right](10{,}000)$

$= v^5\left[{}_5q_{70}\,{}_5q_{80} - {}_4q_{70}\,{}_4q_{80}\right](10{,}000)$

$= \dfrac{10000}{(1.03)^5}\left[\left(1 - \dfrac{l_{75}}{l_{70}}\right)\left(1 - \dfrac{l_{85}}{l_{80}}\right) - \left(1 - \dfrac{l_{74}}{l_{70}}\right)\left(1 - \dfrac{l_{84}}{l_{80}}\right)\right]$

$= 234.82$

141

$$a_{30:40:10} = a_{30:40} - {}_{10}E_{30:40}\, a_{40:50}$$

$$= \left(\ddot{a}_{30:40} - 1\right) - v^{10}\, {}_{10}p_{30}\, {}_{10}p_{40} \left(\ddot{a}_{40:50} - 1\right)$$

$$= (14.2068 - 1) - \left(\frac{1}{1.06}\right)^{10} \left(\frac{l_{50}}{l_{30}}\right)(12.4784 - 1)$$

$$= 13.2068 - \left(\frac{1}{1.06}\right)^{10} \frac{8{,}950{,}901}{9{,}501{,}381}(11.4784)$$

$$= \underline{7.1687}$$

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$$\cancel{(20{,}000\,\ddot{a}_{80:\overline{3}|})(2) - 10{,}000\,\ddot{a}_{80:80:\overline{3}|}}$$

$$\cancel{\ddot{a}_{80:\overline{3}|} = 1 + v(.91) + v^2(.82) = 2.610430839}$$

$$\cancel{\ddot{a}_{80:80:\overline{3}|} = 1 + v(.91)(.91) + v^2(.82)(.82) = 2.398553288}$$

$$\cancel{40000(2.610430839) - 10000(2.398553288) =}$$

$$\cancel{104{,}417.23 - 23\,985.53 = 80\,431.70}$$

142 Cont

$$\mu_{x+s} = \underset{\substack{\text{without}\\ \text{common}\\ \text{shock}}}{0.04} + \underset{\substack{\text{common}\\ \text{shock}}}{0.01} = 0.05$$

$$_tp_x = e^{-\int_0^t 0.05\,ds}$$
$$= \boxed{e^{-0.05t}}$$

(b) $\mu_{y+s} = 0.06 + 0.01 = 0.07$

$$_tp_y = e^{\int_0^t 0.07\,ds} = \boxed{e^{-0.07t}}$$

(c) $\mu_{x+t:y+t} = \mu_{x+t} + \mu_{y+t} + \mu_{cs}$

$$= 0.04 + 0.06 + 0.01 = 0.11$$

$$_tp_{xy} = e^{-\int_0^t 0.11\,ds} = \boxed{e^{-0.11t}}$$

(d) $\mathring{e}_x = \int_0^\infty {_tp_x}\,dt =$

$$\int_0^\infty e^{-0.05t} = \frac{1}{-0.05}\left(e^{-0.05t}\Big|_0^\infty\right)$$

$$= \frac{1}{-0.05}(0-1) = \boxed{20}$$

(e) $\mathring{\ } = \int^\infty e^{-0.07t} = \frac{1}{\ }$

$$\mathring{e}_y = \int_0 \qquad \frac{\quad}{0.07}$$

$$= \boxed{14.286}$$

(f) $$\mathring{e}_{xy} = \int_0^{\infty} {}_tp_{xy}\, dt =$$

$$\int_0^{\infty} e^{-0.11t}\, dt = \frac{1}{0.11} =$$

$$\boxed{9.091}$$

(g) We cannot calculate directly, but can calculate using the box formula.

$$\mathring{e}_x + \mathring{e}_y = \mathring{e}_{xy} + \mathring{e}_{\overline{xy}}$$

$$\mathring{e}_{\overline{xy}} = \mathring{e}_x + \mathring{e}_y - \mathring{e}_{xy}$$

$$= 20 + 14.286 - 9.091$$

$$= \boxed{25.195}$$

(h) $$\bar{A}_{xy} = \int_0^{\infty} v^t\, {}_tp_{xy}\, \mu_{x+t:y+t}\, dt$$

$$= \int_0^{\infty} e^{-\delta t} \cdot e^{-0.11t} \cdot (0.11)\, dt$$

$$= \int_0^{\infty} e^{-(.03+0.11)t}\, (0.11)\, dt$$

$$=$$

$$= \frac{0.11}{0.14} = \boxed{0.78571}$$

(i) $\bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy}$

$$\bar{A}_x = \int_0^\infty e^{-0.03t} \, e^{-0.05t} (0.05)\,dt$$

$$= \frac{0.05}{0.08} = \boxed{0.625}$$

$$\bar{A}_y = \int_0^\infty e^{-0.03t} \, e^{-0.07t} (0.07)\,dt$$

$$= \frac{0.07}{0.10} = \boxed{0.700}$$

$$\bar{A}_{\overline{xy}} = \frac{0.05}{0.08} + \frac{0.07}{0.10} - \frac{.11}{.14} =$$

$$\boxed{0.53929}$$

(143)

$$\mu_{xy}(t) = \mu_x(t) + \mu_y(t) - 0.015$$

$$_tp_{xy} = e^{-\int_0^t \mu_{xy}(s)\,ds}$$

$$= e^{-\int_0^t \mu_x(s)ds} \, e^{-\int_0^t \mu_y(s)ds} \, e^{-\int_0^t -0.015ds}$$

$$= e^{-\int_0^t 0.02s\,ds} \, e^{-\int_0^t (40-s)^{-1}ds} \, e^{\int_0^t 0.015ds}$$

$$= e^{-0.01t^2} \cdot e^{-\ln(40-t)-\ln 40} \cdot e^{0.015t}$$

$$= e^{-0.01t^2} \cdot \frac{40-t}{40} \cdot e^{0.015t}$$

$$= e^{-0.01t^2} \cdot \frac{40-t}{40} \cdot e^{0.015t}$$

$${}_{3|}q_{xy} = {}_3p_{xy} - {}_4p_{xy}$$

$$= e^{-0.01(9)} \cdot \frac{37}{40} \cdot e^{0.045} - e^{-0.01(16)} \cdot \frac{36}{40} \cdot e^{0.06} = \boxed{0.06994}$$

(144) (a) ${}_{20}E_{65}^{m} = v^{20}\ {}_{20}p_{65}^{m}$

$$= \left(\frac{1}{1.08}\right)^{20} \underbrace{\left({}_{20}p_{65}\right)}_{\text{Table}} \underbrace{\left(e^{-(0.01)(20)}\right)}_{\text{common shock}}$$

$$= \left(\frac{1}{1.08}\right)^{20} \left(\frac{2{,}358{,}246}{7{,}533{,}964}\right) \left(e^{-0.2}\right)$$

$$= \boxed{0.05498}$$

(b)

${}_{20}E_{65}^{F} = v^{20}\ {}_{20}p_{60}^{F}$

$$= \left(\frac{1}{1.08}\right)^{20} \underbrace{\left({}_{20}p_{55}\right)}_{\text{Table}} \underbrace{\left(e^{-0.01(20)}\right)}_{\text{common shock}}$$

$$= \left(\frac{1}{1.08}\right)^{20} \left(\frac{5{,}396{,}081}{8{,}640{,}861}\right) \left(e^{-0.2}\right)$$

$$= \boxed{0.10970}$$

(c)

$${}_{20}E^{m\ \ F}_{65:60} = v^{20}\ {}_{20}p^{m\ \ F}_{65:60}$$

$$v^{20}, \underbrace{{}_{20}p_{65}}_{\substack{\text{male}\\ \text{Table}}}, \underbrace{{}_{20}p_{55}}_{\substack{\text{Female}\\ \text{Table}}}, \underbrace{e^{-0.2}}_{\substack{\text{common}\\ \text{SHOCK}}}$$

$$= \left(\frac{1}{1.08}\right)^{20}\left(\frac{2,358,246}{7,533,964}\right)\left(\frac{5,396,081}{8,640,861}\right)\left(e^{-0.2}\right)$$

$$= \boxed{0.03434}$$

(d) Use box formula

$${}_{20}E_{\overline{65:60}}^{m\ \ F} =$$

$${}_{20}E^{m}_{65} + {}_{20}E^{F}_{65} - {}_{20}E^{m\ \ F}_{65:60}$$

$$= 0.05498 + 0.10970 - 0.03434$$

$$= \boxed{0.13034}$$

(145)

$$PV = 20,000\,\ddot{a}_{80:\overline{3}|} + 20000\,\ddot{a}_{80:\overline{3}|} - 10,000\,\ddot{a}_{80:80:\overline{3}|}$$

$$\ddot{a}_{80:\overline{3}|} = 1 + v\,p_{80} + v^2\,{}_2p_{80}$$

$$= 1 + (1.05)^{-1}(.91) + (1.05)^{-2}(.82)$$

$$= 1 + (1.05)^{-1}(.91) + (1.05)^{-2}(.82)$$

$$= 2.61043$$

$$\ddot{a}_{80:80:\overline{3|}} = 1 + v\,{}_1p_{80:80} + v^2\,{}_2p_{80:80}$$

$$= 1 + v^1(.91)^2 + v^2(.82)^2$$

$$= 2.39855$$

$$PV = 40000(2.61043) - 10{,}000(2.39855)$$

$$= \boxed{80{,}431.70}$$

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$$PV = 15{,}000\,\bar{a}_x + 15000\,\bar{a}_y - 5000\,\bar{a}_{xy} + 30000\,\bar{A}_{xy}$$

$$= 15000(\bar{a}_{xy} + \bar{a}_{\overline{xy}}) - 5000\,\bar{a}_{xy} + 30000(1 - \delta\,\bar{a}_{xy})$$

$$= 15000(8 + 10) - 5000(8) + 30000(1 - \ln(1.06)(8))$$

$$= \boxed{246{,}015.46}$$